<u>CONSIDERATIONS FOR THE DESIGN OF CRYSTAL FILTERS</u>

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Abstract: The well established design methods for crystal filters are discussed and compared. Emphasis is placed on predicting the feasibility and the limitation of standard crystal filter designs.

Introduction: The practical realization of frequency selective networks makes it very often necessary to incorporate piezoelectric resonators in the circuit. These resonators, preferably quartz crystals, are used because they represent high-quality resonators of small or reasonable size. To include such elements in a circuit imposes restraints on the design due to three-element structure, the magnitude of the parameters and their influence on other components. The design of networks of this type postulates therefore:

(a) to contemplate circuits which are potentially capable of employing piezoelectric resonators, and
(b) to select related characteristic functions K(s) or transfer-loss functions H(s) such that the performance meets specification

It is frequently not feasible or not practical to substitute piezoelectric resonators in the resulting circuit. To precede a rather complicated circuit calculation by relating the feasibility to the primary design parameters (passband ripple, stopband attenuation and cut-off rate) is therefore economical. Such feasibility studies are possible for some of the typical bandpass realizations of Fig. 1 by

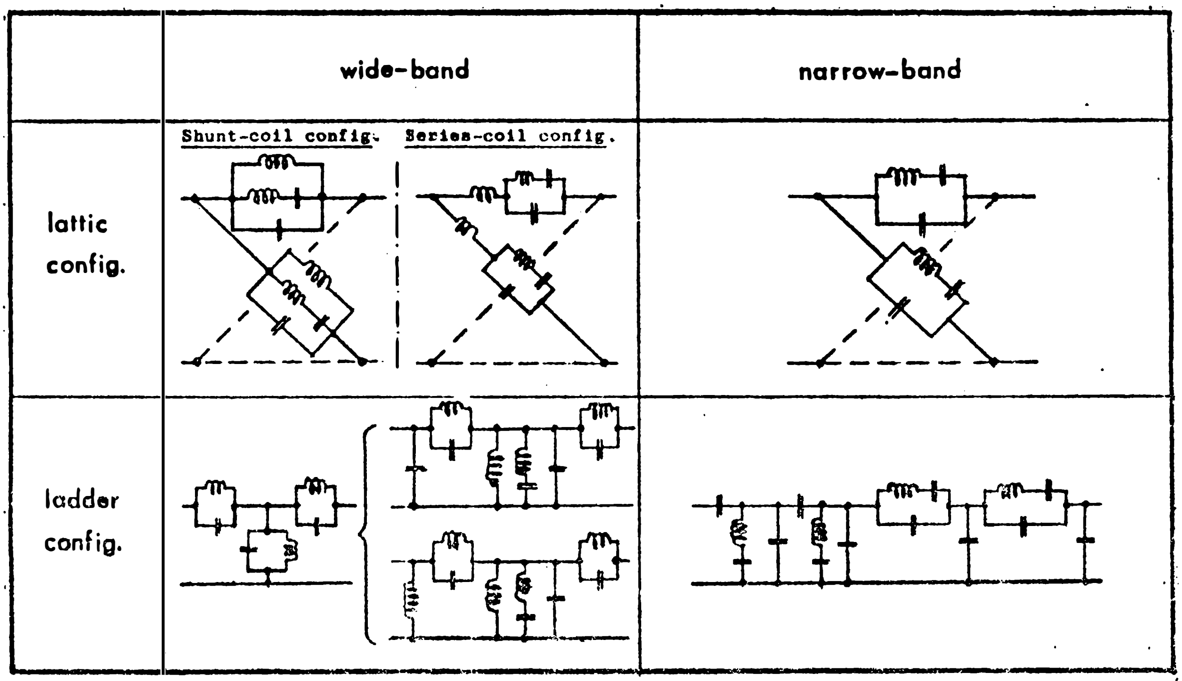


Fig. 1

considering the effect of the pole and zero pattern of the characteristic function K(s) and the transfer-loss function H(s) [SA-1]

$$K(s) = \frac{F(s)}{P(s)} \; ; \qquad H(s) \doteq \frac{E(s)}{P(s)}$$

$$\text{Operating Loss (db)} = \begin{cases} 10 \log_{10} \left(1+|K(s)|^2\right)_{s=jw} \\ \\ 10 \log_{10} |H(s)|^2_{s=jw} \end{cases} \tag{1}$$

E(s) = Hurwitz polynomial ; P(s) = even or odd polynomial

F(s) = Polynomial with real coefficient which must be either even or odd in case of symmetrical or antimetrical filters

A distribution of almost all poles and zeroes of these functions about the center frequency is significant for many crystal filters. Their relative location within this area is very closely proportional to the relative bandwidth. This fact is used in many approximate solutions for the design.

1. Wideband Lattice Filters

The structure of an 8th degree characteristic function K(s) and some general design aspects for shunt-coil configurations are shown in Fig. 2. It is easy to prove that the shunt coils or the shunt capacitors of the two lattice branches become equal whenever a pole of third or higher order exists at zero or infinity, respectively. Analogous

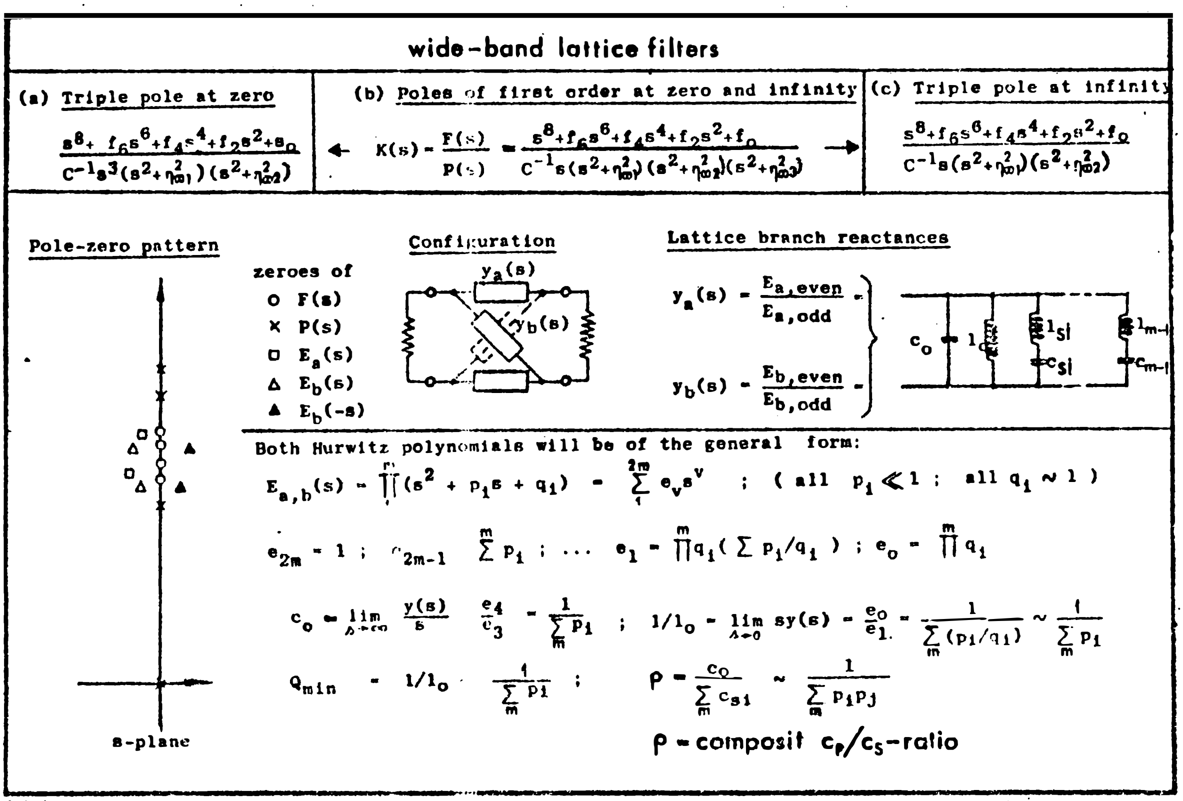
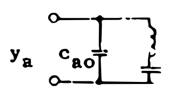
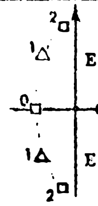
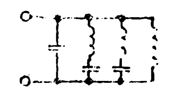
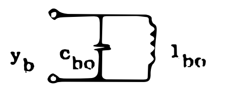
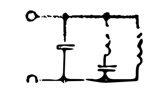
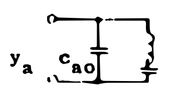
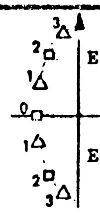
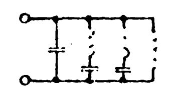
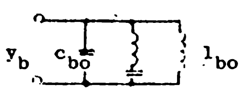
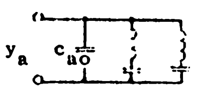
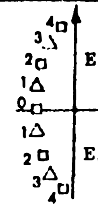
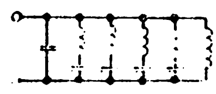
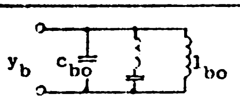
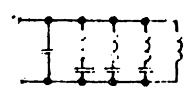


Fig. 2

relations can also be drawn up for the dual series-coil configuration. Limiting factors for either type may be either the Q_{min} of the associated coils or ρ, the composite c_p/c_s-ratio of all crystals. To reduce the quantity "Q_{min}", it is necessary to increase the sum of the real parts of the Hurwitz polynomial $E(s)$. This can be achieved by increasing the complexity (higher degree), by a decrease of the ripple or a combination of both. The composite c_p/c_s ratio on the other hand is improved by moving the zeroes of $E(s)$ closer to the imaginary axis.

An estimate of the limiting quantities Q_{min} and ρ is possible in case of frequency-symmetrical specifications. A major part of the calculations, including the normalized reference circuit, can then be performed in the s-plane of a reference lowpass. By well-known lowpass-bandpass transformations, (Fig. 3), it is possible to express the quantities

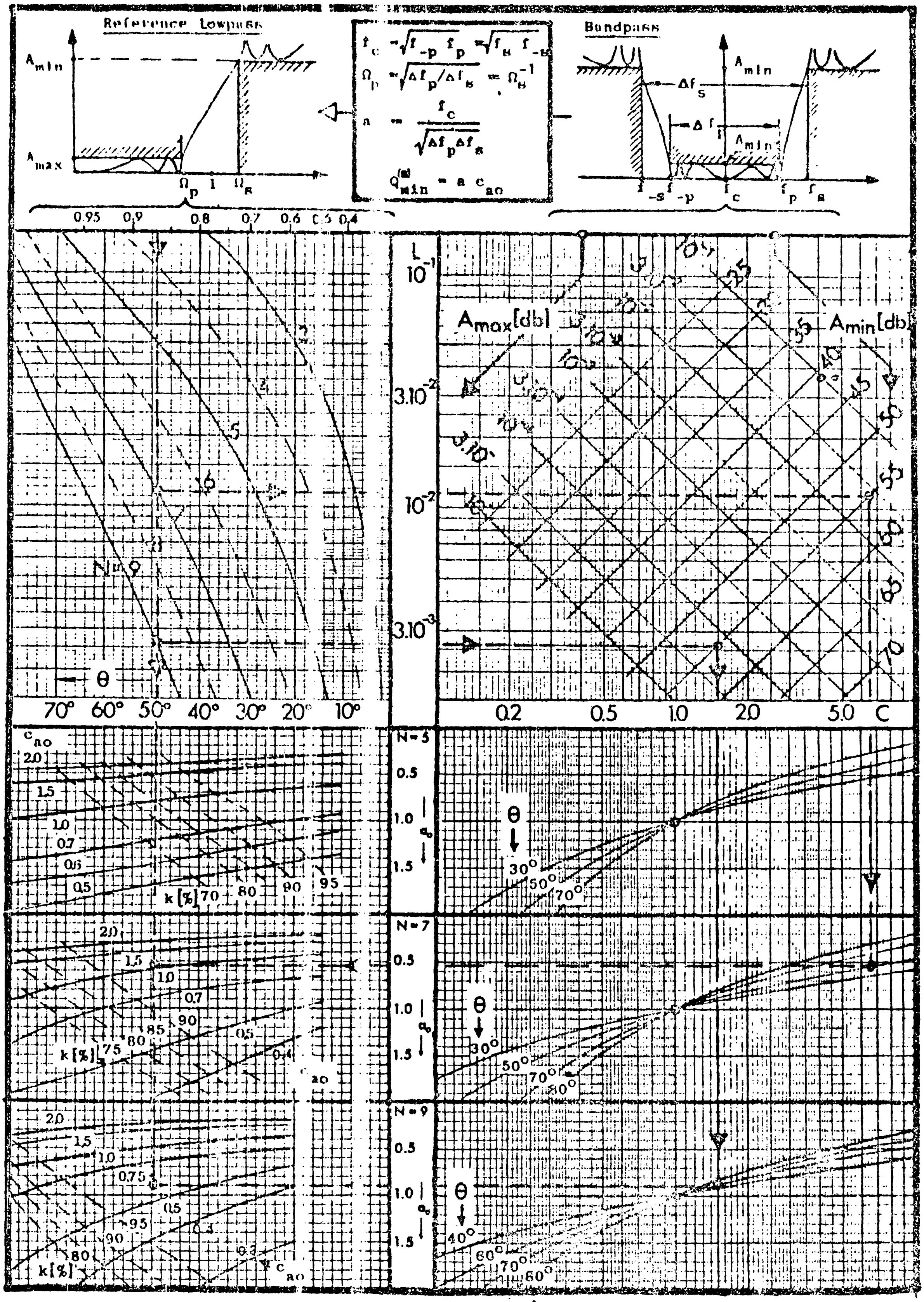
DEGR.	LP-CONFIGUR.		$E_{LP}(s)$		BP-CONFIGUR.
N=5		$c_{ao} = \frac{1}{a_o+p_2}$		$E_a(s) = \{(s+a_o)(s^2+p_2s+q_2)\}$	
		$c_{bo} = \frac{1}{p_1}$ $l_{bo} = p_1/q_1$		$E_b(s) = (s^2+p_1s+q_1)$	
N=7		$c_{ao} = \frac{1}{a_o+p_2}$		$E_a(s) = \{(s+a_o)(s^2+p_2s+q_2)\}$	
		$c_{bo} = \frac{1}{p_1+p_3}$ $l_{bo} = p_1/q_1+p_3/q_3$		$E_b(s) = \{(s^2+p_1s+q_1)(s^2+p_3s+q_3)\}$	
N=9		$c_{ao} = \frac{1}{a_o+p_2+p_4}$		$E_a(s) = \{(s+a_o)(s^2+p_2s+q_2)(s^2+p_4s+q_4)\}$	
		$c_{bo} = \frac{1}{p_1+p_3}$ $l_{bo} = p_1/q_1+p_3/q_3$		$E_b(s) = \{(s^2+p_1s+q_1)(s^2+p_3s+q_3)\}$	

Bandpass transformations (special case: transform. constant "a" $\gg 1$):
(The transform.const. for Cauer Parameter Filters is defined in Fig. 4)

Trasposition of the roots of $E_{LP}(s)$	Reactance Transform. applied to shunt capacitors
$s_v^{(lp)} = a_v + jb_v \longrightarrow s_v^{(BP)} \sim \frac{a_v}{2a} \pm j\left(1 + \frac{b_v}{2a}\right);$	$\left.\begin{matrix} c_{ao}^{(BP)} = a\, c_{ao}^{(LP)} \\ c_{bo}^{(BP)} = a\, c_{bo}^{(LP)} \end{matrix}\right\}$ therefore $\left\{ \boxed{\begin{matrix} Q_{a,min} = a\, c_{ao}^{(LP)} \\ Q_{b,min} = a\, c_{bo}^{(LP)} \end{matrix}} \right.$

Fig. 3

Q_{min} and ρ as functions of the Hurwitz polynomial $E_{LP}(s)$ and the transformation constant "a". For many standard lowpass filters, the zeroes of $E_{LP}(s)$ form well defined patterns. The nomograph of Fig. 4 represents the evaluation of such a pattern for the particular case of Cauer Parameter filters. The formulas on top of this Figure and the following numerical example may serve to demonstrate the procedure to estimate the quantity "Q_{min}":



Reference Lowpass
Bandpass
Amin
Amax
Ωp 1 Ωs
fc = √(f-p fp) = √(fs f-s)
Ωp = √(Δfp/Δfs) = Ωs^-1
n = fc / √(Δfp Δfs)
Q(m)min = n cao
Δfs
Δfp
f-s f-p fc fp fs
0.95 0.9 0.8 0.7 0.6 0.5 0.4
L
10^-1
3.10^-2
10^-2
3.10^-3
Amax[db]
Amin[db]
N=9
θ
70° 60° 50° 40° 30° 20° 10°
0.2 0.5 1.0 2.0 5.0 C
N=5
N=7
N=9
cao
k[%]

Fig. 4

Bandpass Specifications:

Passband: $f_{-p}=16.000$ Mc, $f_{+p}=16.120$ Mc $\Delta f_p = 120$ kc

Stopband: $f_{-s}=15.980$ Mc, $f_{+s}=16.140$ Mc, $A_{min} = 55$ db $\Delta f_s = 160$ kc

According to the formulas of Fig. 4 (slide rule accuracy), these specifications yield first:

$$f_c \; 16.060 \text{ mc}; \; a=116 \; ; \; \Omega_p = 0.866 \; ; \; \theta=50^o$$

and subsequently by means of the nomographs:

N = 5	(excessive A_{max})		
N = 7	$A_{max} = 0.05$ db	$c_{ao} = 1.25$	$Q_{min}^{(a)} = 128$
N = 9	$A_{max} = 10^{-4}$ db	$c_{ao} = 0.60$	$Q_{min}^{(a)} = 69$

The capacitor "c_{bo}" which is responsible for $Q_{min}^{(b)}$ can be expressed in terms of c_{ao}:

$$c_{bo} = k \, c_{ao}$$

$$\text{thus:} \quad Q_{min}^{(b)} = k \, Q_{min}^{(a)} \qquad (2)$$

The family of curves "k(%)" is included in the nomograph.

Also included are the cut-off curves for antimetrical lowpass filters N = 4, 6, 8. They are useful to interpolate the Q_{min} of quasi-frequency-symmetrical bandpass filters of 8th, 12th and 16th degree for which an actual reference lowpass does not exist. They require a total of 2, 4, or 6 crystals, respectively, when realized in half-lattice configuration.

The feasibility limits of wideband crystal filters will be exceeded whenever a large transformation constant "a" causes an unrealistic Q_{min} of the shunt or series coils. A realization may then be possible by crystal-capacitor filters (either in lattice or ladder configuration).

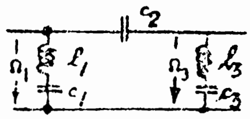
The feasibility limits may also be exceeded in the opposite sense when the transformation constant "a" becomes too small for a realizable composite c_p/c_s ratio. However, this ratio is proportional to "a^2" and only bandpass filters with considerably large bandwidth will be effected. In this case it is very often possible to satisfy the specifications by other means.

2. Wideband Ladder Filters

A steep rise of the attenuation at the edge or edges of a passband requires very often impractical values of the Q for those resonant circuits which are responsible for the closest attenuation poles. Piezoelectric resonators can be substituted for these critical circuits if the structure permits such a substitution. Network transformation similar to those shown in Fig. 5 may be employed to achieve such suitable structure. The transformation "4" of this Figure, originally suggested

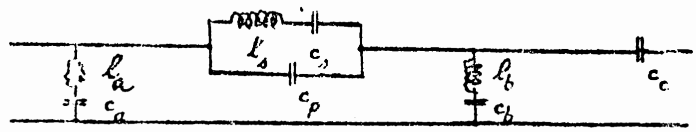
ORIGINAL CIRCUIT	TRANSFORMED CIRCUIT

1



$$c = \frac{1}{\ell\,\Omega_1^2} \qquad \ell = \frac{1}{c\,(\Omega_3^2-\Omega_1^2)}$$

$$A = c_1 - c \qquad L = \frac{\ell\,\ell_1}{\ell-\ell_1} \qquad M = \frac{\ell\,\ell_3}{\ell+\ell_3}$$



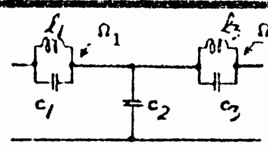
$c_a = A$, $\ell_a = L$

$\ell_s = \dfrac{1}{c_s\,\Omega_1^2}$, $c_s = c\,(1+\ell_3/\ell)$, $c_p = c_2(1+\ell_3/\ell)$, $c_p/c_s = \Omega_1^2/(\Omega_3^2-\Omega_1^2)$

$\ell_b = M$, $c_b = \dfrac{1}{\ell_b\,\Omega_1^2}$

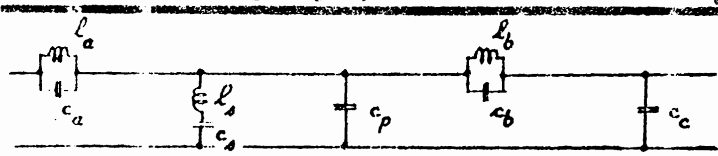
$c_c = c\,(1+\ell/\ell_3)$

2



$$c = \frac{\Omega_3^2}{\Omega_1^2-\Omega_3^2}\,c_2 \qquad \ell = \frac{1}{c\,\Omega_1^2} \qquad L = \ell_1 - \ell$$

$$A = \frac{c\,c_1}{c-c_1} \qquad B = \frac{c\,c_2}{c+c_2+c_3} \qquad C = \frac{c\,c_3}{c+c_2+c_3} \qquad D = \frac{c_2\,c_3}{c+c_2+c_3}$$



$c_a = A$, $\ell_a = L$

$\ell_s = \dfrac{1}{B(\Omega_1^2-\Omega_3^2)}$, $c_s = \dfrac{1}{\ell_s\,\Omega_3^2}$, $c_p = B$, $c_p/c_s = \Omega_3^2/(\Omega_1^2-\Omega_3^2)$

$c_b = C$, $\ell_b = \dfrac{1}{c_b\,\Omega_1^2}$

$c_c = D$

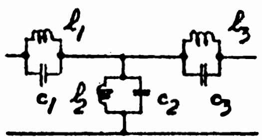
3 and 4

$\eta_1 = (\ell_1 c_1)^{-1/2}$

$\eta_2 = (\ell_2 c_2)^{-1/2}$

$\eta_3 = (\ell_3 c_3)^{-1/2}$

$\boxed{\eta_2 < \eta_3 < \eta_1}$ → 3



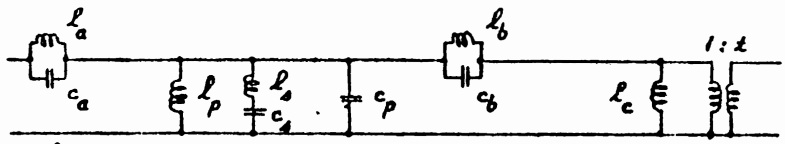
$\boxed{\eta_1 < \eta_3 < \eta_2}$ → 4

$$c = \frac{\eta_3^2-\eta^2}{\eta_1^2-\eta_3^2}\,c_2 \qquad \ell = \frac{1}{c\,\eta_1^2} \qquad A = \frac{c\,c_1}{c-c_1} \qquad L = \ell_1 - \ell$$

$$M = (\ell+\ell_2)[1 + c_2/(c_2+c)]$$

$$B = \frac{c\,c_2}{c+c_2+c_3} \qquad C = \frac{c\,c_3}{c+c_2+c_3} \qquad D = \frac{c_2\,c_3}{c+c_2+c_3}$$

3



$\ell_a = L$, $c_a = A$

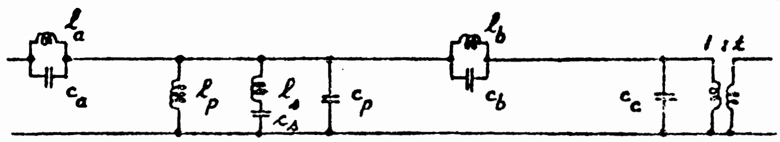
$\ell_s = \dfrac{\eta_3^2/B}{(\eta_3^2-\eta_2^2)(\eta_1^2-\eta_3^2)}$, $\ell_p = \dfrac{t\,M}{t + D\,M\,\eta_1^2}$, $c_p = B + D/t$, $c_s = \dfrac{1}{\ell_s\,\eta_3^2}$

$c_p/c_s = \dfrac{\eta_3^4\,[1 + c_3/(c+c_2)]}{(\eta_3^2-\eta_2^2)(\eta_1^2-\eta_3^2)}$

$\ell_b = \dfrac{1}{c_b\,\eta_1^2}$, $c_b = C/t$

$t = 1 + c_2/c$, $\ell_c = \dfrac{t^2\ell_3/D\eta_1^2}{\ell_3\eta_2^2/\eta_1^2 - 1/D\eta_2^2}$

4



$\ell_a = L$, $c_a = A$

$\ell_s = \dfrac{\eta_3^2/B}{(\eta_3^2-\eta_2^2)(\eta_1^2-\eta_3^2)}$, $\ell_p = \dfrac{t\,M}{t + D\,M\,\eta_2^2}$, $c_p = B + \dfrac{D/\eta_2^2}{t\,\eta_1^2}$, $c_s = 1/\ell_s\eta_3^2$

$c_p/c_s = \dfrac{\eta_3^4\,[1 + c_3\eta_2^2/(c\,\eta_1^2 + c_2\eta_2^2)]}{(\eta_3^2-\eta_2^2)(\eta_1^2-\eta_3^2)}$

$\ell_b = \dfrac{1}{c_b\,\eta_1^2}$, $c_b = C/t$

$t = 1 + \dfrac{c_2\eta_2^2}{c\,\eta_1^2}$, $c_c = \dfrac{D\,(1-\eta_2^2/\eta_1^2)}{t^2}$

Fig. 5

by Poschenrieder ([PO-1]), and the transformation "3" are especially useful. The resulting structures prevent a signal flow at the pole frequency by a low impedance shunt at the highly stable series resonance of the crystal. Furthermore, the parallel coil permits a suitable transformation of the crystal impedance. The structure of all transformed circuits of Fig. 4 and others can also result initially by a proper realization procedure ([HA-1]).

It should also be mentioned that narrow bandstop filters can also be employed to supplement the performance of conventional LC-filters ([SA-2]).

3. Narrow-band Lattice and Ladder Filters

The performance of a conventional narrow bandpass filter will deteriorate considerably due to the inherent losses of actual components, especially coils. Structures which can be realized by crystals and capacitors only are then of advantage. Significant for lattice and ladder configurations of this type are pairs of zeroes of K(s) and H(s) on the real axis of the s-plane in addition to those in the immediate vicinity of the passband. This fact was first pointed out by Saal [SA-2] for lattice filters and by Colin [CO-1] for ladder filters. The pertinent function K(s) is then:

$$K(s) = \frac{(s^2 - a_0^2)}{(1 + a_0^2)s} K_0(s) = \frac{(s^2 - a_0^2)}{(1 + a_0^2)s} \underbrace{C \prod_{\nu} \frac{(s^2 + \eta_{0\nu}^2)}{(s^2 + \eta_{\infty\nu}^2)}}_{K_0(s)} \tag{3}$$

All attenuation poles must occur at finite frequencies in case of genuine ladder configuration. Part or all of them may be transposed to zero or infinity if lattice configurations are contemplated.

The function $K_0(s)$ may be considered to be the characteristic function of an antimetrical bandpass. The poles and zeroes of this function are almost solely responsible for the performance in the passband and its vicinity. Cauer's q-functions are a convenient means to design a $K_0(s)$ which provides a Chebyshev variation within the passband limits for any arbitrary set of attenuation poles ([CA-1], pgs 548-560)

$$K_0(s) = \sqrt{10^{0.1 A_{max}} - 1} \; \frac{q^2(s) + 1}{q^2(s) - 1} \tag{4}$$

where the "composite q-function q(s)" can be calculated from the "individual q-functions $q_v(s)$" which are related to the specified attenuation poles:

$$\frac{q(s) + 1}{q(s) - 1} = \prod_{\nu} \frac{q_v(s) + 1}{q_v(s) - 1} \quad ; \quad q_v(s) = m_v \sqrt{\frac{s^2 + 1/\tau}{s^2 + \tau}}$$

$$\tau = \sqrt{f_p / f_{-p}} \quad ; \quad m_v = \sqrt{\frac{\eta_{\infty v}^2 - \tau}{\eta_{\infty v}^2 - 1/\tau}} \tag{5}$$

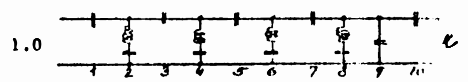
The antimetrical function $K_0(s)$ is then supplemented by the symmetry factor "$(s^2 - a_0^2)/(1 + a_0^2)s$" which provides for a first order pole each at zero and infinity. Its magnitude is close to unity in the immediate vicinity of the passband.

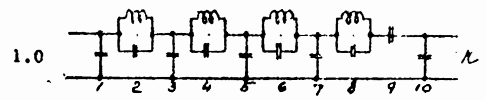
The value "a_o" is of significance for the composite c_p/c_s ratio of either branch. A rather accurate estimate of these quantities is possible in the case of frequency-symmetrical performances for which the pertinent Hurwitz polynomials $E_a(s)$ and $E_b(s)$ of the bandpass can be related to the polynomials $E_{LP}^{(a)}(s)$ $E_{LP}^{(b)}(s)$ of an antimetrical reference lowpass. (See Fig. 3). The even-degree cut-off curves of Fig. 4 and the formulas in the top part of this figure permit the selection of a suitable table reference lowpass and the calculation of the transformation constant "a". Expressed in terms of the real parts "a_v" of $E_{LP}^{(a)}(s)$ or $E_{LP}^{(b)}(s)$, the quantity   becomes:

$$\rho = \frac{c_p}{\sum\limits_i c_{si}} = \frac{a}{(a_o + 1/a_o)} \quad \frac{1}{\sum\limits_v a_v} \qquad (6)$$

For a selected reference lowpass and a specified transformation constant, this quantity becomes a maximum where $(a_o + 1/a_o)$ is a minimum which occurs for $a_o = 1$.

The well-known realization of lattice structures follows a procedure analogous to the one shown in Fig. 2. Crystal capacitor ladder structures, however, require special attention. Due to the factor $(s^2 - a_o^2)$ in the characteristic function, the open-and the short-circuit impedance





	z_{open}	z_{shunt}
$c_1 =$	1.73006	1.73008
$\ell_2 =$	168.398	168.405
$c_2 =$	0.00595	0.00595
$c_3 =$	2.61987	2.61991
$\ell_4 =$	17.8752	17.8781
$c_4 =$	0.05607	0.05606
$c_5 =$	26.1639	26.1552
$\ell_6 =$	1.18146	1.18005
$c_6 =$	0.84835	0.84936
$c_7 =$	409.336	409.817
$\ell_8 =$	0.10428	0.10431
$c_8 =$	9.61105	9.60876
$c =$	3820.21	6612.72
	$= c_9$	$= c_9 + c_{10}$
r	0.0006196	

Open and shunt impedances:

$$z_{open} = \frac{e_7 s^6 + e_5 s^4 + e_3 s^2 + e_1}{2s^7 + (e_6 + f_6)s^5 + (e_4 + f_4)s^3 + (e_2 + f_2)s}$$

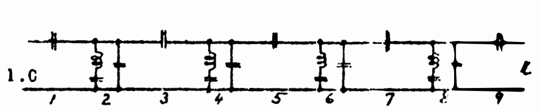
$$z_{shunt} = \frac{(e_6 - f_6)s^6 + (e_4 - f_4)s^4 + (e_2 - f_2)s^2 + (e_0 - f_0)}{e_7 s^7 + e_5 s^5 + e_3 s^3 + e_1 s}$$

$$F(s) = s^8 + f_6 s^6 + f_4 s^4 + f_2 s^2 + f_0$$

$$E(s) = s^8 + e_7 s^7 + e_6 s^6 + e_5 s^5 + e_4 s^4 + e_3 s^3 + e_2 s^2 + e_1 s + e_0$$

$f_0 = - e_0$ due to the zero on the real axis

z_{open}	z_{shunt}
0.576120	0.576120
0.005906	0.005906
168.9161	169.9161
0.38316	0.38316
0.05534	0.05534
18.0282	18.0282
0.038614	0.038614
0.836242	0.836242
1.193093	1.193093
0.002470	0.002470
9.40665	9.406659
0.106065	0.106065
0.000153	0.000266
$= c_{9,10}$	$= c_9$
1592	



$c_1 = 1.730$

$\ell_2 = 168.0$ $c_2 = 0.00595$ $\eta_2 = 0.998855$ $c_{p2} = 1.766$ $\rho = 297$

$c_3 = 0.854$

$\ell_4 = 168.4$ $c_4 = 0.00595$ $\eta_4 = 0.998855$ $c_{p4} = 1.488$ $\rho = 250$

$c_5 = 0.713$

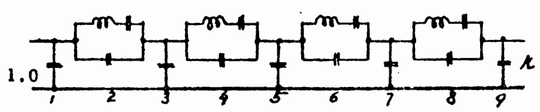
$\ell_6 = 168.4$ $c_6 = 0.00595$ $\eta_6 = 0.998855$ $c_{p6} = 1.488$ $\rho = 250$

$c_7 = 0.854$

$\ell_8 = 168.4$ $c_8 = 0.00595$ $\eta_8 = 0.998855$ $c_{p8} = 1.766$ $\rho = 297$

$c_9 = 1.730$

$r = 1.000$



$c_1 = 0.576$

$\ell_2 = 480.5$ $c_2 = 0.00208$ $\eta_2 = 1.002905$ $c_{p2} = 0.592$ $\rho = 289$

$c_3 = 1.079$

$\ell_4 = 480.7$ $c_4 = 0.00208$ $\eta_4 = 1.00306$ $c_{p4} = 0.544$ $\rho = 261$

$c_5 = 1.189$

$\ell_6 = 480.7$ $c_6 = 0.00208$ $\eta_6 = 1.00306$ c_{p6} 0.544 $\rho = 261$

$c_7 = 1.079$

$\ell_8 = 480.5$ $c_8 = 0.00208$ $\eta_8 = 1.002905$ $c_{p8} = 0.592$ $\rho = 284$

$c_9 = 0.576$

$r = 1.000$

Fig. 6

at either terminal pair is one degree lower than the degree of the characteristic function. Consequently, the ladder development of either impedance will leave one capacitor undetermined. (See Fig. 6).

A convenient means to overcome any ambiguity of the last capacitors is to realize the circuit in shunt-and open-circuit condition. The comparison of the values of all other elements can be used as a criterion for adequate precision of the calculation. The numerical examples of Fig. 6 are related to a 10.7 Mc filter with all poles at 13.5 Kc above or below the passband, respectively. The left circuit was calculated with 24, the right circuit with 28 significant digits. As the comparison of "shunt" and "open" shows, 24 significant digits were just adequate.

Any realization of ladder structures will normally yield the configurations of crystal resonators only if the attenuation poles above and below the passband alternate. Any other sequence will require subsequent network transformation similar to those indicated at the bottom of Fig. 6. These particular transformations were performed to yield equal inductance for all the crystals of one structure.

The lowest c_p/c_s ratio within the final structure can also be estimated without an elaborate calculation by means of the following formula:

$$\rho = \sqrt{\frac{f_c}{\Delta f}} \tag{7}$$

in which f_c is the center frequency and Δf the interval between the lowest (or highest) attenuation pole and the upper (or lower) passband limit. For the left hand circuit of Fig. 6, the lowest pole was at 10.685 Mc and the upper passband limit 10.703 Mc. Equation (7) yields then ρ = 243, sufficiently close to the actual value.

4. Lattice-Ladder Hybrid Structures

Lattice configurations are a special case of structures which provide more than one path for the transmission of the signal. Attenuation poles occur at those frequencies at which cancellation of the energy flow takes place. This is an inherent disadvantage whenever high stopband attenuations are specified. It is therefore often convenient to cascade several sections in tandem and to employ lattice configurations only whenever this particular structure offers advantages, for instance better c_p/c_s ratio. Three different approaches are, at present, in use.

(a) Cascading of identical lattice configurations. According to Fig. 7, the stopband attenuation is more than doubled. The passband widens due to additional zeroes of the overall characteristic function. The passband ripple is considerably increased.

(b) Bisection of a symmetrical network. The priority to the solution goes to H. Piloty [PT - 1]. A different solution, developed by H. Adler and E. H. Bradley ([GU - 1], pg. 467), was recently applied by Szentirmai [SZ - 1] to lattice configurations. According to Piloty's method, it is first necessary to augment the denominator polynomial of an overall characteristic function K(s) by factors which make it a perfect square. The more factors required to achieve this the less economical will be the overall structure. A realization of a network by bisection is therefore recommended only in case of coinciding attenuation poles.

(c) Lattice transformations applied to suitable sections of an established ladder structure. This solution was first indicated by Szentirmai [SZ - 1] and later supplemented by J.A.C. Bingham [BI - 1]. A different approach is presented in Fig. 8. To achieve optimum conditions of the final circuit, it is useful to realize the initial ladder circuit with an appropriate sequence of pole frequencies. (See the numerical example).

CASCADED LATTICE FILTERS

$H_1(s)$: $K_1(s)$ — Loss: A_1(db)	$H_2(s)$; $K_2(s)$ — Loss: A_2(db)	$\equiv$	$H(s)=K_1(-s)\ K_2(s)+H_1(s)\ H_2(s)$ — $K(s)=H_1(-s)\ K_2(s)+K_1(s)\ H_2(s)$

Special case : identical networks

$$K(s) = 2\,K_1(s)\,\frac{E_{1,odd}(s)}{P_1(s)}$$

$$H(s) = K_1(-s)\,K_1(s) + H_1^2(s)$$

Approximation of the performance:

passband : $|K(s)| \sim 2\,|K_1(s)| + |K_1(s)|^3$

Stopband : $A(db) \sim 2\,A_1(db) + 6$ db

Numerical example : Crystal-capacitor bandpass of 6th degree

Center frequ. f_c = 456 kc; passband limits : f_c ±140 cps ; p = 25 %

attenuation poles: f_c ± 6 kc ; A_{min} = 56 db

These specifications can be satisfied by a lattice related to the following characteristic function $K_1(s)$:

$$K_1(s) = \frac{F_1(s)}{P_1(s)} = \frac{(s^2+\Omega_{02}^2)(s^2+\Omega_{01}^2)(s^2-1)}{C^{-1}s\,(s^2+\Omega_{\infty 1}^2)(s^2+\Omega_{\infty 2}^2)} = \frac{s^6+1.000000154s^4-1.000000156s^2-0.999999998}{(3.00435s^5+6.010766s^3+3.004356s).10^{-3}}$$

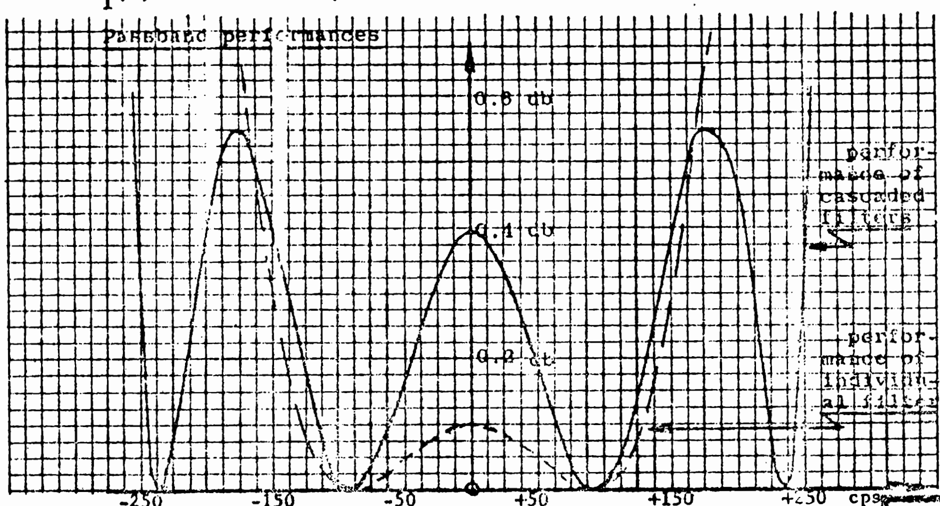
Hurwitz Polynomial :

$$E_1(s) = s^6+2.001330026s^5+3.002656577s^4+4.002662126s^3+3.002656575s+ 2.001330022s + 1.00000000$$

$$E_{1,odd}(s) = 2.001330026s^5+4.002662126s^3+2.001330022s$$

Overall characteristic function K(s) of the tandem connection :

$$K(s) = \frac{F_1(s)\,E_{1,odd}(s)}{P_1^2(s)} = \frac{(s^2+\Omega_{01}^2)(s^2+\Omega_{02}^2)(s^2-1)(2.001330026s^4+\ldots.\ 2.001330022)}{s(3.00435s^4+6.010766s^2+3.004356)^2 10^{-6}}$$



f_c=456 kc

Fig. 7

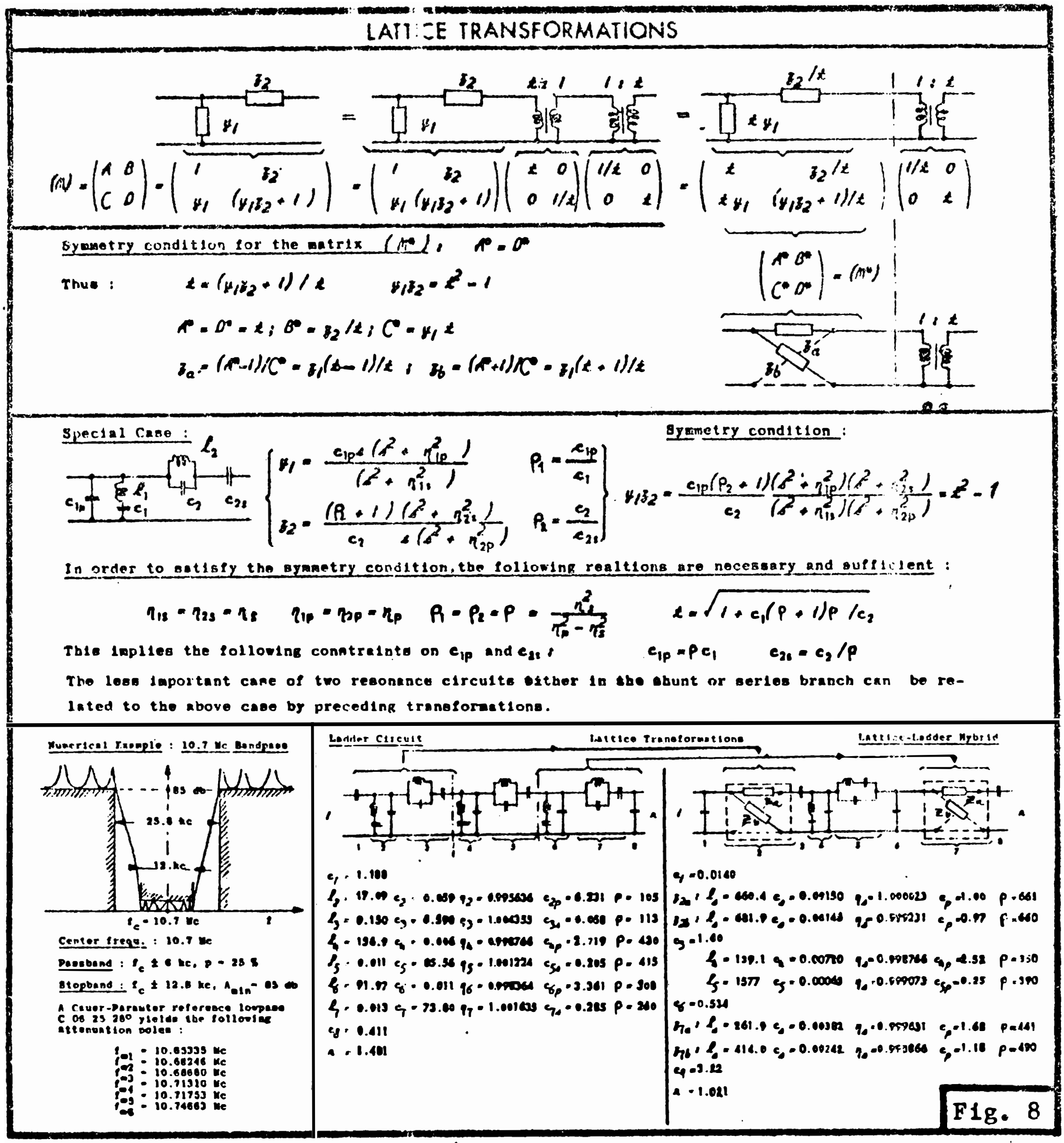


Fig. 8

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